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Dynamic Pricing Strategy with Internal Reference Effect and Competition

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Abstract
This paper studies the integrative impact of reference effect and substitution effect on the price decision of firms. We start with a monopolist selling the product in two sequential selling periods. An internal reference effect as an additional component of the base demand occurs in the second period. Analytically, such effect can encourage the firm to set a higher price firstly to better establish a higher reference effect which exaggerates the demand in a near future. We explicitly illustrate the price distortion between the scenario with and without reference effect and an upper bound of discount percentage within the two periods. Next, we study the duopoly scenario in which substitution effect among multiple firms coexists with reference effect. Interestingly, we find that the two effects have distinct impacts on the price of the product. Concisely, competition effect affects the price in a vertical way, where the price base (traditional price without reference effect) is decreasing in competition intensity, and reference effect affects the price in a lateral way, where the additive component of the price is increasing (decreasing) in reference intensity in the first (second) period. Overall, the competition will reduce a firm's profit. However, the reference effect will increase a firm's profit.

Keywords: reference effect; competitive intensity; dynamic pricing; two periods; discount percentage.

INTRODUCTION

The gradual growth of multiple sales channels and internet-based marketing has stimulated widespread attempts with dynamic pricing, a practice of varying prices for the identical goods over selling horizons or across customer classifications with the expected purpose for firms to increase revenue in response to competitive situations. In many settings, various price practices enable firm better control of the demands and revenues in various selling horizons by allocating capacity to different price classes over time. Observingly, reference effect can be traced in various pricing practices where current price is suggested to make references on previous prices of the product or homogeneous product by other firms. Initiated by Monroe (1973), the effects of reference price on consumer choice have been widely accepted as an empirical generalization in marketing field. Greenleaf (1995) and Kopalle et al. (1996) consider reference price effects in modeling competitive behavior of firms and provide managerial insights for firm and manufactures. According to reference price effects on customer behavior, Lilien and Yoon (1990) investigate changes in price elasticity of demand along the product life cycle. Fibich et al. (2005) derive an expression for the price elasticity of demand in the presence of reference price effects with the variance of consumer evaluations and conclude that the effect of reference price is most noticeable immediately after a price change, before consumers have had time to adjust their reference price. Gavious and Lowengart (2012) aim at exploring such effect on demand and study firms' pricing strategies when asymmetric effects of reference price exists over a finite or infinite decision horizon.

In addition, we attempt to extend prior research on competition among multiple firms. Within the last decade, a significant body of researches in dynamic pricing context has been developed concerning with better modeling of customer behavior. Ovchinnikov and Milner (2005) study the effects of customer response over selling periods. Liu and van Ryzin (2008) study two-period pricing models and use quantity decision as means to induce early purchases. Cachon and Swinnery (2009) discuss a markdown pricing strategy in a two-period model and find that quick response offers more value in the presence of strategic customers. They find that customers update their expectations about the probability of acquiring an item at a discount based on previous interaction with the seller, while taking less into account the role of competition, or seldom considering an explicit competition among multiple firms. A monopolistic market is usually assumed to justify whether a reasonable approximation of the effects of competitor response can be captured by a price-sensitive demand model. As Netessine and Shumsky (2005) depicts, a possible reason could be that even relatively simple models lack pure strategy equilibrium, making them difficult to extend to more complicated situations.

However, pure monopoly rarely exists in real situation and it is desirable to explicitly capture the effects of competition between firms on their pricing policies. Potential competitors may behave strategically by enabling a lower price to attract customers in various selling periods, which intensifies the complexity and diversity of the competition between the firms.

The objective of this article is to investigate how the firm makes a two-period price decision with the existence of reference effect. That is, if the demand of the product is influenced by reference effect, how do the prices in two periods interact with each other? What is the optimal pricing strategy of the firm? In addition, if price competition among multiple firms is considered, what is the integrative influence of reference effect and substitution effect? Next, mathematical models are proposed to tackle the above issues.

Model

Consider duopoly firms selling a homogeneous product over two periods. The product is produced or procured at a constant unit cost, c. The demand of a firm depends on three factors, his selling price, the price of competitor, and his historical price.

The base demand is characterized by a traditional lineal function, $d = a - \beta p$, where a is market potential, and β is coefficient of his selling price.

Competition (substitution) effect: since the product is homogeneous, the demand of a firm is increasing in his competitor's price. We call this a substitution effect. The impact of competitor is $\gamma(p_j - p_i)$, where γ is coefficient of substitution effect.

Reference effect: the influence of historical price on demand can be characterized as reference effect. In marketing practice, price reduction can attract the customer since he can perceive additional surplus of the valuation and an additional transaction utility. In our two-period model, given p_1 and p_2 as the price in period 1 and 2 respectively, the reference effect can be characterized by $\phi(p_1 - p_2)$, where ϕ is coefficient of reference effect.

In the second period, the consumer not only has the basic valuation on the product, but also will get an additional transaction utility from price reduction because of the reference effects (see Thaler, 1985). Here, we assume that a consumer's transaction utility from reference effect is linear to the price reduction, i.e., $\phi(p_{i,1} - p_{i,2})$, where ϕ is a parameter measuring the consumer's sensitivity to price change.

In addition, we make the following assumptions:

A1 (perfect information assumption): Both firms have perfect information.

A2: The firm's historical price and competitor's price has less effect than his current selling price. That is, $\beta > \gamma$ and $\beta > \phi$.

A3: There is no discount between two periods.

Also for simplicity, we normalize the cost c to zero. The reason lies in that, as an exogenous parameter, the cost only impacts the deterministic payoffs in which c appears in the form of $a - \beta c$. This means that a linear part of the production cost will not affect the incentives of the firms for the demand and profit.

Next, we first consider a benchmark model where a monopoly sells the product in two periods.

BENCHMARK MODEL: MONOPOLY SCENARIO

In this section, we refer firm i as the monopoly firm.

In the first period, the profit of firm i is:

$$\pi_{i,1}^{M} = p_{i,1}(a - \beta p_{i,1}); \qquad (1)$$

where superscript M refers to monopoly scenario.

In the second period, taking into account the reference effect incurred by first-period price, the profit of firm i in period 2 is:

$$\pi_{i,2}^{M} = p_{i,2}(a - \beta p_{i,2} + \phi(p_{i,1} - p_{i,2})); \qquad (2)$$

The objective for firm i is to maximize the total profit in two selling periods,

$$\pi_i^{M} = \max_{p_{i,1}, p_{i,2}} (\pi_{i,1}^{M} + \pi_{i,2}^{M}) \qquad (3)$$

We obtain the optimal solution by backward induction, which is proposed in Theorem 1.

Theorem 1 *In the monopoly scenario, the optimal pricing decision of firm i is given by:*

$$\begin{cases} p_{i,1}^{M*} = \dfrac{(2\beta + 3\phi)a}{4\beta^2 + 4\beta\phi - \phi^2} \\ p_{i,2}^{M*} = \dfrac{(2\beta + \phi)a}{4\beta^2 + 4\beta\phi - \phi^2} \end{cases} \qquad (4)$$

Next, some discussions on the optimal pricing decision are presented in the following Corollaries and Propositions.

Corollary 1.1 *In the monopoly scenario, the price in period 2 is lower than that in period 1.*

Noting from Theorem 1 that, the prices setting can be described as simple formulas illustrated as $p_{i,1}^{M*} = \frac{a}{2\beta}(1 + \Phi_1)$ and $p_{i,2}^{M*} = \frac{a}{2\beta}(1 - \Phi_2)$, where $\Phi_1 = \dfrac{\phi(2\beta + \phi)}{4\beta^2 + 4\beta\phi - \phi^2}$ and

$\Phi_2 = \dfrac{\phi(2\beta-\phi)}{4\beta^2+4\beta\phi-\phi^2}$. Since $p_{i,1}^{M*} = p_{i,2}^{M*} = \dfrac{a}{2\beta}$, when $\phi = 0$, the additive form of the prices in Theorem 1, Φ_1 and Φ_2, can be regarded as the extent of reference effect and its influence on the price is explicitly shown. Set $k = \phi/\beta$ which represents the relative responsiveness of reference effect to price elasticity, and it satisfies that $0 < k < 1$. Then we have $\Phi_1 = \dfrac{k(2+k)}{4+4k-k^2}$ and $\Phi_2 = \dfrac{k(2-k)}{4+4k-k^2}$. It is easy to prove that $\Phi_1 > \Phi_2 > 0$.

Corollary 1.1 implies that the firm will choose a price reduction form in accordance with the existence of reference effect. In fact, there are two possible incentives for the firm to have a price reduction during the selling periods. Firstly, the firm can capture more customers' surplus by utilizing a pricing policy similar to price skimming. Secondly, he can successfully form a higher reference price which forms additional valuation of the customers on account of reference effect. As a result, more demand is stimulated in later period.

In order to consider the influence of reference effect, we divide monopoly scenario into two cases. Case *MR* refers to the case with the reference effect, and case *MN* refers to the one without.

Corollary 1.2 *In the first period, the selling price under case MR is higher than that under case MN. However, in the second period, the selling price under case MR is lower than that under case MN.*

Corollary 1.2 illustrates that, to take advantage of the reference effect, the firm intentionally raises the selling price in the first period while lowers the selling price in the second period with the desire to amplify the price gap. This result differs from prior research (see Greenleaf 1995, Kopalle et al. 1996 and Fibich et al. 2003 for example) in that our model predicts a high-low price structure.

Proposition 1.1 *Under case MR,*

(i) In the first period, the optimal selling price is increasing in ϕ*;*

(ii) In the second period, the optimal selling price is decreasing in ϕ *when* $\phi \le 2\left(\sqrt{2}-1\right)\beta$ *and is increasing in* ϕ *when* $\phi > 2\left(\sqrt{2}-1\right)\beta$.

Intuitively, one may think that the first-period price is increasing in ϕ, and the second-period price is decreasing in ϕ. The proposition shows a result different from such intuition. Proposition 1.1 indicates that, in the first period, the stronger the reference effect, the higher the selling price will be. However, in the second period, the selling price is decreasing in ϕ when ϕ is relative small. When ϕ exceeds a threshold the selling price is increasing in ϕ, instead. Such threshold depends on β. That is, the strength of reference effect and self price effect has combined influence on the price gap. A possible explanation is that, given the coefficient of reference effect approaches price elasticity, the decrease of the profit motivated by demand decrease is moderated by the increase of the profit motivated by price increase.

In an attempt to better illustrate the price change between case MR and case MN, define $T_{i,j}^{P} = \left|\dfrac{p_{i,j}^{MR} - p_{i,j}^{MN}}{p_{i,j}^{MN}}\right|$ as price percentage distortion of firm i in period j, where $p_{i,j}^{MR}$ and $p_{i,j}^{MN}$ are optimal selling price under case MR and MN respectively. Since $p_{i,1}^{M*} = \dfrac{a}{2\beta}\left(1+\Phi_1\right)$ and $p_{i,2}^{M*} = \dfrac{a}{2\beta}\left(1-\Phi_2\right)$, we can easily find that $T_{i,1}^{P} = \Phi_1$ and $T_{i,2}^{P} = \Phi_2$.

Proposition 1.2 *In the first period, the price percentage distortion satisfies that* $0 < T_{i,1}^{P} < \dfrac{3}{7}$. *In the second period, the price percentage distortion satisfies that* $0 < T_{i,2}^{P} < \dfrac{2-\sqrt{2}}{4}$.

Proposition 1.1 and 1.2 describes the fact that, in the first period, although optimal selling price increases with the strength of reference effect, the price percentage distortion under various cases can not exceed to a threshold, i.e. $\dfrac{3}{7}$. Correspondingly, since in the second period, optimal selling price decreases with the strength of such effect, the price percentage distortion can not exceed to a threshold, i.e. $\dfrac{2-\sqrt{2}}{4}$, either. An obvious reason is that the strength of reference effect cannot surpass that of self price effect.

Since the high-low price structure stems from reference effect, one may wonder how the price decision in different periods varies with the strength of reference effect, ϕ. Correspondingly, we make a further effort to illustrate how the strength of reference effect affects the price gap between the

periods. Define $\Delta p \equiv p_{i,1}^{M*} - p_{i,2}^{M*}$ as the gap of optimal selling price between two periods, we have the following proposition.

Proposition 1.3 *Under case MR,* Δp *is increasing in* ϕ.

Proposition 1.3 implies, when the reference effect is intensified, the optimal selling price gap between two periods always increases, no matter whether the price in the second period decreases or increases. That is to say, when ϕ exceeds the threshold, the increase of first-period price is always larger than the increase of second-period price, given a small addition of ϕ.

From the point view of consumers, percentage discount is another important index to evaluate the deal. Define $\delta_i^{M*} = \dfrac{p_{i,1}^{M*} - p_{i,2}^{M*}}{p_{i,1}^{M*}}$ as the percentage discount. It is obvious that $\delta_i^{M*} = \dfrac{2k}{2+3k}$, which implies that the stronger the reference effect, the larger percentage discount is offered. Since $0 < k < 1$, we can obtain proposition 1.4.

Proposition 1.4 *Under case MR, the percentage discount is increasing in* ϕ *and does not exceed* 40%*, i.e.* $0 < \delta_i^{M*} < 40\%$.

Proposition 1.4 implies that, in comparison with the first period, the price will have a relatively reduction in the second period. Since the consumers can expect a price reduction in the second period, those who are not time sensitive may wait and make a purchase in the later period.

COMPETITION: DUOPOLY MARKET

Section 2.1 makes a detailed discussion about the influence of reference effect on the price and profit in a monopoly market. Although the monopoly market may differ from the real market in practice, we can also clearly realize how and to what extent reference effect works and give preferable managerial insights to the firm as well. Since widespread existence of the competition in retailing market, we focus our further attention on the competition effect among the firms and its interaction with reference effect.

In this section, we consider a duopoly market in which two firms exist, indexed by $i \in \{1,2\}$ and $j = 3 - i$, and compete with each other on prices and market shares. The game between the duopoly can be described as:(i) In the first period, firm i and firm j set price $p_{i,1}$ and $p_{j,1}$ simultaneously; (ii) In the second period, the two firms set price $p_{i,2}$ and $p_{j,2}$ simultaneously. The goal of a firm is to maximize his profit by the price decisions in two periods.

Denote γ as competition intensity of the firms. In general, since competition effect does not exceed self price effect, we assume $\gamma < \beta$. Besides, denote $\dfrac{\gamma}{\beta}$ as the relative responsiveness of competition intensity to self price effect. Obviously, $\gamma/\beta < 1$.

The profits of the firms in two periods can be defined as:

$$\pi_{i,1}^D = p_{i,1}\left(a - \beta p_{i,1} + \gamma\left(p_{j,1} - p_{i,1}\right)\right), i, j = 1,2, i \neq j$$

$$\pi_{i,2}^D = p_{i,2}\left(a - \beta p_{i,2} + \gamma\left(p_{j,2} - p_{i,2}\right) + \phi\left(p_{i,1} - p_{i,2}\right)\right), i, j = 1,2, i \neq j$$

and the goal of firm i is to maximize his total profit:

$$\pi_i^D = \max_{p_{i,1}, p_{i,2}} \left(\pi_{i,1}^D + \pi_{i,2}^D\right), i, j = 1,2, i \neq j$$

where superscript D refers to duopoly scenario.

In order to consider the influence of reference effect, we divide duopoly *scenario* into two cases. Case *DR* refers to the case with the reference effect, and case *DN* refers to the one without. In either case, competition effect always makes its influence on the duopoly.

We obtain the optimal solution by backward induction, which is proposed in Theorem 2.

Theorem 2. *In the duopoly market, the equilibrium pricing decision of firm* i *is given by:*

$$\begin{cases} p_{i,1}^{D*} = \dfrac{(t+2m)(t+2\phi)^2 + \phi(t+m+\phi)^2}{t(t+2\phi)^2(t+2m) - \phi^2(t+m+\phi)^2} a \\ p_{i,2}^{D*} = \dfrac{(t+2m)(t+2\phi)(t+\phi)}{t(t+2\phi)^2(t+2m) - \phi^2(t+m+\phi)^2} a \end{cases} \tag{8}$$

where superscript D *refers to duopoly scenario, and* $t = 2\beta + \gamma$, $m = \gamma + \phi$.

Next, we will discuss the optimal decision on price in a duopoly market and make a detailed comparison with that in a monopoly market. The results are presented in the following Corollaries and Propositions.

Corollary 2.1 (two periods) *Under case DR, the price in period 2 is lower than that in period 1.*

Define

$$\Delta p_i^{D*} = \frac{\phi\left[(t+2m)(t+\phi) + (t+m+\phi)^2\right]}{t(t+2\phi)^2(t+2m) - \phi^2(t+m+\phi)^2} a$$

as the price gap between the two periods. Since $\Delta p_i^{D*} / \partial\phi > 0$, Δp_i^{D*} is increasing in ϕ. When $\phi = 0$, the price gap is zero. It implies that Δp_i^{D*} is always positive under case DR. This is similar to case MN. That is, price reduction is always the optimal policy for the firms when reference effect exists, no matter whether competition effect exists or to what extent its strength is.

In a better effort to depict the influence of competition effect, we define the price base as the price in various scenarios without reference effect. That is, the price base in the monopoly scenario (resp. duopoly scenario) is defined as the price under case MN (resp. case DN). Correspondingly, the change of price base can well illustrate how and to what extent the competition effect influences the price in various scenarios.

Corollary 2.2 (price base) *The price base under case MN is larger than that under case DN.*
Since the price under case DN (resp. case MN) is $\frac{a}{2\beta}$, the price base in the monopoly scenario (resp. duopoly scenario) is $p_B^M = \frac{a}{2\beta}$ (resp. $p_B^D = \frac{a}{2\beta+\gamma}$). Obviously, $p_B^D < p_B^M$ given γ is positive. Corollary 4.2 implies that competition effect has a negative influence on the price base. This gives a vivid illustration of the common sense that competition effect forces the duopoly to decrease their prices.

Corollary 2.3 (price base in γ) *The gap ratio of price base in duopoly scenario to monopoly scenario is increasing in γ.*

Define $\Delta_B = \frac{p_B^M - p_B^D}{p_B^M}$ as the gap ratio of price base in duopoly scenario to monopoly scenario. Correspondingly, $\Delta_B = \frac{\lambda}{2+\lambda}$ where $\lambda = \gamma/\beta$. Given β is constant, the larger γ, the larger λ, the larger the gap ratio. That is, the stronger the competition effect, the lower the price base decreases.

Corollary 2.4 (reference effect) *In the first period, the price under case DR is higher than that under case DN (price base). In the second period, the price under case DR is lower than that under case DN (price base).*

Under case DN, the equilibrium price in two periods is $p_{i,1}^{DN*} = p_{i,2}^{DN*} = \frac{a}{2\beta+\gamma}$ when $\phi = 0$, where superscript DN refers to no reference effect. If reference effect occurs,

$$p_{i,1}^{D*} > \frac{(t+2m)(t+2\phi)^2}{t(t+2\phi)^2(t+2m)} a = \frac{a}{t} = \frac{a}{2\beta+\gamma}$$

and

$$p_{i,2}^{D*} < \frac{(t+2m)(t+2\phi)(t+\phi)-(t+m+\phi)^2}{t(t+2\phi)^2(t+2m)-t(t+m+\phi)^2} a = \frac{a}{t} = \frac{a}{2\beta+\gamma}.$$

Thus, corollary 4.4 is proved.

Next, we will figure out how and to what extent competition effect and reference effect impact the price decision of the duopoly.

Proposition 2.1 (Influence of γ) *The equilibrium price of the duopoly under case DR is lower than that under case MR in two periods.*
Since $\partial p_{i,j}^{D*} / \partial\gamma < 0$, then $p_{i,j}^{D*}$ is decreasing in γ and reach its maximum when $\gamma = 0$. Since $p_{i,j}^{M*} = p_{i,j}^{D*}$ given $\gamma = 0$, that is, $p_{i,j}^{M*} \geq p_{i,j}^{D*}$.

Proposition 2.2 (Influence of ϕ) *Under case DR,*

(i) in the first period, the equilibrium price is increasing in ϕ;

(ii) in the second period, the equilibrium price is decreasing in ϕ when $\phi < \phi^T$ and is increasing in ϕ when $\phi > \phi^T$, where ϕ^T is the threshold of ϕ. In addition, ϕ^T is increasing in ϕ/γ. Especially, given β is the same as under case MN, ϕ^T is larger than $2(\sqrt{2}-1)\beta$, no matter how γ values.

Proposition 2.3 *Under case DR, reference effect and competition effect have distinct influences on the equilibrium prices of the firms. Specifically, competition effect influences the equilibrium price in a vertical way. Especially, the larger reference effect, the lower the price base is. Correspondingly, reference effect influences the equilibrium price in a lateral way. Especially, the larger reference effect, the larger the price gap in two periods is.*

From the price formulation, we can divide the price into two parts and separate ϕ and γ in order to make a clear description of the influence of two effects on price decision. First, let $\gamma = 0$, and the price in Theorem 4 is simplified as in case MN. Further, let $\phi = 0$, and the price is simplified as

$p_{i,1}^{D*} = p_{i,2}^{D*} = \frac{a}{2\beta}(1 - R(\gamma))$, where $R(\gamma) = s/(2+s)$. Since $\partial R(\gamma)/\gamma < 0$, the optimal prices are decreasing in γ no matter whether reference effect exists.

Moreover, the reference effect widening the price variance, the competitive effect just get the price close to the cost, Therefore, their combined effect is that price reduction is always the optimal policy for the firms when the reference price effect exists, no matter whether competition effect exists concurrently. The difference between the two situations lies in the extent variance of the price reduction.

Corollary 2.5 *Under case DR, the discounting factor by the firm i satisfies $\delta_i^{D*} < \delta_i^{M*}$. Specially,*

(a) $0 < \delta_i^{D*} < 40\%$ *when* $1 > \phi > \gamma > 0$;

(b) $0 < \delta_i^{D*} < 33.65\%$ *when* $0 < \phi < \gamma < 1$

Since $\delta_i^{D*} = \frac{k(8 + 7s^2 + 16ks + 8k^2 + 16s + 16k)}{(2 + 3s + 2k)(2 + s + 2k)^2 + 4k(1 + s + k)^2}$, then δ_i^{D*} is decreasing in s and reaches its maximum when $s = 0$. Thus, $\delta_i^{D*} = \delta_i^{M*} = \frac{2k}{2 + 3k}$.

This is an interesting phenomenon. If the coefficient of reference effect exceeds that of competition effect, the price percentage off will not exceed 40%, which is the same as that under case MR. Contrarily, if the coefficient of competition effect exceeds that of reference effect, the upper bound of price percentage off will decrease to 33.65%, which is lower than 40%. It is consistent with the fact that competition between the firms will induce a lower price in the first period, which results in a smaller price reduction in the second period. In other word, competition effect mitigates the price gap initiated by reference effect during selling periods.

CONCLUDING MARKS

We dealt with the problem of dynamic pricing of the firms in two sequential selling periods, particularly, focusing our attention on two effects, reference effect and competition effect, and their integrative effects on price decision of the firms. We emphasize on an internal reference effect where the first-period price has a reference effect on the second-period price. And we start with a simplified scenario where a monopolist sells the product in two sequential selling periods. The monopoly firm initially charges a price in the first period but internal reference effect occurs in the second period. We extend conventional demand function and the effect is formalized as an additional component of the base demand of the firm and its impact can be explicitly shown as well. Analytically, such effect can encourage the firm to set a higher price firstly to better establish a higher reference price and to pose a relatively lower price in the second period so as to aggrandize the impact of the effect. We explicitly illustrate the price distortion between the scenarios with and without reference effect and an upper bound of discount percentage within the two periods. Despite the fact that competition is of a subjective characteristics of the real market, we study the duopoly scenario in which substitution effect among multiple firms coexists with reference effect. Interestingly, we find that the two effects have distinct impacts on the price of the product. Concisely, competition effect affects the price in a vertical way and reference effect affects the price in a lateral way.

We conclude main contributions of this paper as follows. First, while most previous literature formulates the reference effect with a market-level model and characterizes its features in a statistical way, we model it in an individual firm level and derive its valuation to the firm's market share and profit so as to explore the insightful managerial intuitive for his performance. Second, we examine integrative impacts of reference price and competitive effect on price decision of the firm and give an illustrative representation of their inner relationship. Third, several interesting results have been well illustrated in the analysis which are different from pervious researches or their valuable extensions, such as distortion and discount percentage, etc. The results will make a helpful guidance for the firm on account of percentage discount or other promotion practice in multiple selling periods.

Limitations and possible extensions of this paper are as follows. Firstly, we do not consider the discount factor for the profit during the two periods. Secondly, we only consider simplified scenarios where a monopolist or duopoly firms exist in a market, and ignore the quality diversity of homogeneous product by various firms. Thirdly, it is would be interesting to incorporate the behavior of consumers into the model. We will make further explorations on these issues in future.

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